LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2024**

PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 13-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	Define a first-order differential equation and give an example.
b)	Define the Wronskian of $x_1(t), x_2(t)$, and $x_3(t)$.
c)	Differentiate solution matrix and fundamental matrix.
d)	Define regular singular point.
e)	Give an example of a non-oscillatory equation and explain it.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	Let $R=\{(t,x):\|t\|\le 1, \|x\|\le 1\}$ and $f(t,x)=1+2tx^2$, $(t,x)\in R$. Find the Lipschitz constant. (i) 3 (ii) 4 (iii) 5 (iv) 6
b)	Which of the following intervals e^t and e^{-t} are linearly independent? (i) $(-\infty,0)$ (ii) $(0,\infty)$ (iii) $(-\infty,\infty)$ (iv) all the intervals
c)	Let Φ be a fundamental matrix for the system $x'=A(t)x$. Then $\Phi(t+s)=$¿ (i) Φ¿ (ii) Φ¿ (iii) Φ¿ (iv) Φ¿
d)	What is the value of Bessel function $J_0(0)$? (i) 0 (ii) 1 (iii) $\frac{1}{2}$ (iv) ∞
e)	Which of the following is true regarding the relationship between the zeros of solutions in Sturm's separation theorem? (i) The zeros of two independent solutions must coincide (ii) One solution must have twice as many zeros as the other (iii) The zeros of two independent solutions alternate (iv) The zeros of one solution lie between those of the second solution, but not vice versa
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Use Picard's successive approximation method to find the solutions of $x'=-x$, $x(0)=1$, $t\ge 0$.
4	Apply the Wronskian to check the linear independence of the following functions. (i) $\sin t, \cos t$ on $I=R$, (ii) $t^2, t\|t\|$ on $I=(-1,1)$, (iii) $e^{\alpha t}\cos\beta t, e^{\alpha t}\sin\beta t$ on $I=$¿ R.
5	Transform the equation $x''-2x'+x+0$ with $x(0)=0$ and $x'(0)=1$ into a system of equations and solve it.
6	Using Legendre polynomials, show that $\int_{-1}^{1} P_n(t)P_m(t)\,dt=0$ whenever $m\ne n$
7	Use the statement, "If a solution of the first differential equation has a certain known property P, then the

	solution of a second differential equation has the same or a related property P under certain hypothesis" to support a mathematical result.

SECTION C – K4 (CO3)	
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Analyze the solution of the equation $x' + a(t)x = b(t)$ where *a* and *b* are known continuous function defined on the interval *I* and hence, find the solution of $(1+t)^2 x' - x = 0$, $t \in ¿$.
9	Derive the various possible solutions of a second-order linear homogeneous differential equation with constant coefficients.
10	Let $A : I \rightarrow M_n(R)$ be continuous. Suppose a matrix Φ satisfies the system $x' = A(t)x$, evaluate $(\det \Phi)'$.
11	Let *a* be continuous and positive on $(0, \infty)$ with $\int_t^{\infty} a(s)ds = \infty$. Assume that *x* is any non-zero solution of $x'' + a(t)x = 0$ existing for $t \geq 0$. Prove that *x* has infinite zeros in $(0, \infty)$.
SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 15 = 15)**
12	Explain the method of variation of parameters for solving the second order differential equation $x''(t) + b_1(t)x'(t) + b_2(t)x(t) = h(t)$ and determine the solution for $x'' - \frac{2}{t}x' + \frac{2}{t^2}x = t \sin t$, where $t \in ¿$.
13	Discuss the Hille-Wintner comparison theorem.
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14	Develop a mathematical model for an arms competition between two nations by considering various possible scenarios.
15	Construct two linearly independent solutions of the equation $t^2 x'' + t x' + (t^2 - p^2)x = 0$ when *p* is not an integer and discuss what happens to these two solutions when *p* is an integer.

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